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MOTION OF THERMICS IN A STRATIFIED ATMOSPHERE

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Regular features of motion of thermics in a stratified atmosphere are considered. Thermics are single and multiple free volumes of floating gas, which successively arise with a certain frequency near a horizontal surface. In a real atmosphere, large-scale thermics of this type appear, for example, as a result of powerful pulse actions on the ambient medium that are successively produced at one point near the Earth surface.

1. We consider the following problem. Let n thermics, which are axisymmetric free volumes Q_i filled by a medium with density $\rho_0 = \rho_0(z)$, successively arise at times $t_i = \tau i$ ($i = 0, 1, 2, \ldots, n-1$) in the field of the force of gravity g in a medium with density $\rho_i < \rho_0(0)$ bounded from below by a horizontal plane S. The boundary Q_i is the surface F_i, and a part of this surface can coincide with S at $t = t_i$. We introduce a cylindrical coordinate system (z, r) whose z axis coincides with the axis of symmetry of the problem and is directed upward and whose origin is located at the point of intersection of the z axis with the surface S. It is required to determine the motion of the gas that initially filled the thermics in the space above the plane S at $t > 0$.

Previous model experiments on interaction of thermics [1] showed that both the interaction process and the character of further ascending of the thermics are unstable: even comparatively small variations of the initial position and size of the thermics can significantly alter the regular features of motion of the resulting objects. Stratification of the atmosphere aggravates this situation.

The method of physical modeling is rather beneficial for predicting the motion of interacting thermics in a real atmosphere.

It is shown [2–6] that, under the conditions considered (high Reynolds numbers, absence of boundary layers), the motion of thermics is determined by buoyancy and inertial forces, whereas the effect of viscous forces is relatively small. In considering the motion of a thermic in a homogeneous medium, the scale factor becomes inessential; in a stratified medium, the scale ratio of the phenomenon and stratification is an additional parameter determining the motion [6–8]. The presence of viscosity cannot be neglected in the case of comparatively slow development of motion (for example, in the case of thermic hanging and destruction at the level of zero buoyancy).

Lessen [9] considered the method of qualitative and quantitative modeling of free flows with very large differences in Reynolds numbers and the methods of modeling of stratification phenomena.

A physical model of the atmosphere can be constructed using nitrogen–helium and freon–air mixtures. By varying their composition, we can obtain gases with densities from $0.13\rho_0$ to $5\rho_0$, where ρ_0 is the density of air. Stratification is generated by maintaining constant compositions of the mixtures in two or three horizontal cross sections of a hermetically sealed chamber, for example, by a slow supply of the mixtures from the top and bottom of the chamber and their exhaustion from a horizontal slot in the central part of the chamber.

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The maintenance of a correct flow rate should be automated. Owing to natural diffusion, density stratification is established in the test section of the chamber. The density gradients are determined by the initial densities of the mixtures and by the distances from the top and bottom of the chamber to the slot. This method of maintaining stratification can be called *dynamic*, in contrast to the static method used in experiments conducted in water solutions. The necessity of using the *dynamic* method in a gaseous medium is caused by the fact that the diffusion coefficients in gases are significantly greater than in liquids.

We consider the motion of an incompressible medium with density stratification and compare it with the motion of a compressible atmosphere.

We assume that the parameters of state (temperature T_0, pressure P_0, and density ρ_0) in an undisturbed atmosphere depend only on the height z above the Earth surface: $T_0 = T_0(z)$, $P_0 = P_0(z)$, and $\rho_0 = \rho_0(z)$. At all points of space, these parameters are related by the equation of state $P_0 = \rho_0 R T_0$ and the hydrostatic equation $dP_0/dz = -\rho_0 g$.

Following [3, 9, 10], we assume that, when the particles that constitute the atmosphere (microvolumes filled by air) move vertically, their state changes adiabatically: the particles neither lose nor gain any heat. In this case, stratification of the atmosphere is characterized by the potential temperature $\theta_0(z)$:

$$\theta_0 = T_0(1000/P_0)^{0.286}$$

(the pressure P_0 here is measured in millibars).

The relationship of the quantities θ_0, C_0, and ρ_0 is determined as [9]

$$-1/\theta_0(d\theta_0/dz) = 1/\rho_0(dP_0/dz) + g/C_0^2, \tag{1}$$

where C_0 is the speed of sound.

If the density of the particles changes rather slowly in the course of motion, the buoyancy effects related to stratification of a compressible atmosphere can be studied on the basis of results obtained for an incompressible liquid [9, 10]. In this case, passing to atmospheric conditions, we should replace the density $\rho(z,t)$ (with account of the sign of the gradient) by the potential temperature $\theta(z,t)$. As follows from the Euler equations and (1), this substitution means the loss of the acoustic solution, which is irrelevant in many problems.

We note that, when large-scale thermics ascend to high altitudes in a real atmosphere, their volume and density change because of the decrease in the atmospheric pressure. In interpretation of the modeling results, we should take into account that it is not possible to ensure model conditions adequate to full-scale conditions in terms of the density of the thermic during its ascent.

Thus, a real atmosphere with known constraints can be put into correspondence with an incompressible medium with density stratification whose motion can be modeled in laboratory conditions. The Brunt–Väisälä frequency

$$N = \left(\frac{g}{\rho_0}\left|\frac{d\rho_0}{dz}\right|\right)^{1/2} \tag{2}$$

enters the criterial relationships as the main governing parameter.

2. It is shown [5, 6, 11, 12] that all single thermics with density ρ_i floating up in a homogeneous medium with density ρ_0 are similar in the character of motion and internal structure. The explicit dependence of the equations of motion and boundary conditions on the initial radius $R_{0i} = (3Q_i/(4\pi))^{1/3}$ and relative difference in density $\xi_i = (\rho_i - \rho_0)/\rho_0$ is eliminated by introduction of the dimensionless functions and coordinates

$$t^0 = t\left(\frac{g\xi_i}{R_{0i}}\right)^{1/2}, \qquad Z^0 = \frac{Z}{R_{0i}}, \qquad U^0 = \frac{U}{(g\xi_i R_{0i})^{1/2}}, \tag{3}$$

where t is the time, Z is the length, and U is the velocity.

It follows from (3) that, within the framework of the adopted assumptions, the similarity is ensured by geometrical similarity of the initial configurations of the thermic and proportionality of the density ratio. For similar thermics, we can introduce dimensionless functions and coordinates that allow us to avoid consideration of the scales of the phenomenon.

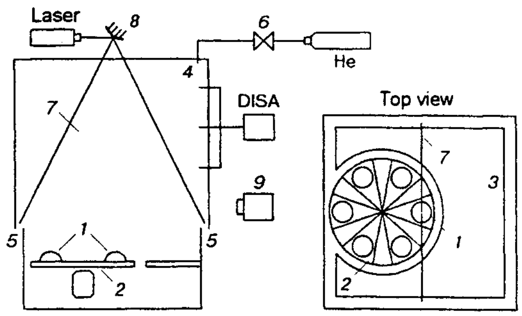

Fig. 1

In modeling the motion of multiple thermics (with equal initial densities), apart from the geometrical similarity, we must ensure the correspondence of times and coordinates of appearance of the thermics under full-scale and model conditions.

In the case of a stratified atmosphere, additional governing parameters appear. One of them can be the height of zero buoyancy of the thermic H_0 [the level of equilibrium for which we have $\rho_0(H_0) = \rho_i$].

3. We estimate the effect of stratification on the height of thermic ascending. As noted in [6], up to $z \sim 5R_0$, the thermic is a floating-up sphere. The energy equation for such a sphere is

$$\int_0^z [\rho_0(z) - \rho_1(z)]Q(z)g\,dz = (1/3)\pi\rho_0 R^3 u^2 = \rho_1(z)Q(z)\int_0^z \xi(z)/(1-\xi(z))\,dz, \tag{4}$$

where $\rho_1(z)Q(z) = \rho_1(0)Q_0 = \text{const}$ is the mass of the light gas.

The velocity of an ascending cloud is

$$u = 2/3(\xi g R)^{1/2}. \tag{5}$$

Substituting (5) into Eq. (4), we obtain $(R^0)^4 = (1/9)\xi_1(1-\xi)\rho_1(0)/(\xi(1-\xi_1)\rho_1)z^0$, where $\xi_1/(1-\xi_1)$ is some mean value of $\xi/(1-\xi)$ on the interval $(0, z)$. From here, we have

$$\frac{dz^0}{dt} = \sqrt{3}\xi/\xi(0)(\xi_1(1-\xi)\rho_1(0)/(\xi(1-\xi_1)\rho_1))^{1/8}z^{1/8} = k\sqrt{3}z^{1/8}.$$

The value of $\xi(z) = (\rho_0(z) - \rho_1(z))/\rho_0(z)$ in a real atmosphere changes by 10–15% with change in height from 0 to 9 km, the ratio $\rho_1(0)/\rho_1(z)$ changes less than twice, and the coefficient k determining the effect of stratification up to the height mentioned is $1 \leqslant k \leqslant 1.1$. These variations of the coefficient k do not exert a significant effect on the height of the ascent.

4. To model the motion of large-scale thermics, a facility was manufactured (Fig. 1) whose test section contained mechanisms for obtaining a programmed spatial-temporal location of the thermics.

The thermics appeared when a nitrogen-helium mixture of a given composition (prescribed density) colored by tobacco smoke was set free from under a soap film. The periodicity of appearance of the thermics in the vicinity of one point in space was ensured by means of successive supply of "charges" 1 located on a rotating platform 2 to a given space of the test section of the chamber 3, where the thermic "initiation" occurred: the buoyancy volume was set free by means of burning through the soap film. It is essential that the "charge" stopped at a given place 0.05–0.1 sec prior to initiation and remained motionless until the starting moment. Thus, several successive "explosions" were performed with equal time intervals of 0.25–0.4 sec.

This facility allows one to study the motion of thermics in a stratified medium. Stratification is achieved by using a nitrogen-helium mixture of variable composition as the model of atmosphere. The density distribution over the height is determined by natural diffusion of helium and by the velocity of helium injection to the upper part of the chamber through orifice 4. When helium slowly filled the upper part of the test section (from the top to cross section 5, which has slots connecting the chamber with the atmosphere), the velocity

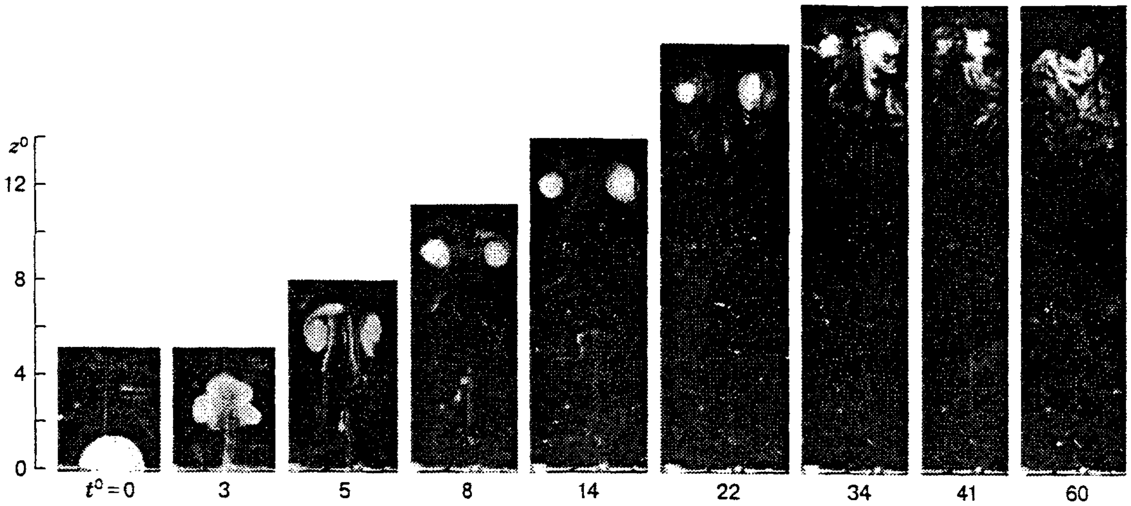

Fig. 2

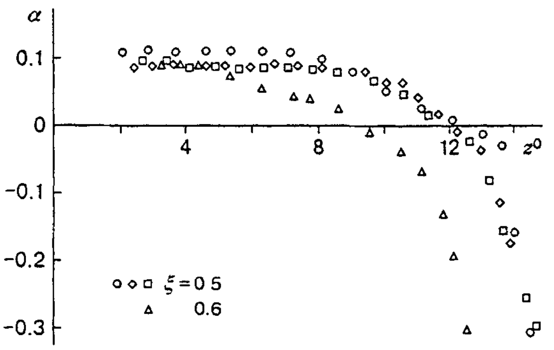

Fig. 3

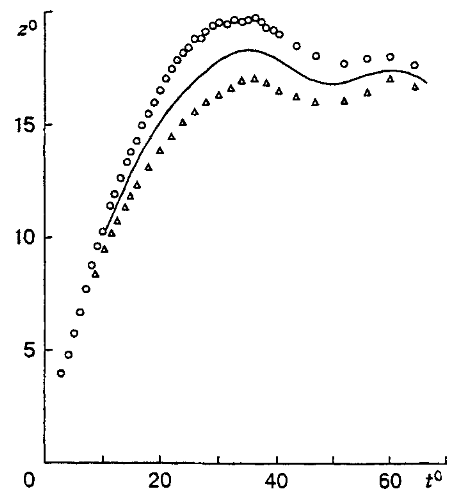

Fig. 4

of its injection was reduced almost to zero using reducer 6. The mixture in the test section was retained in this state for a certain time for natural stratification to establish. As a result, the upper part 4 of the hermetically sealed test section 3 contained almost pure helium, and the lower part 5 contained air. The density of the medium in the chamber at different levels was controlled by a DISA hot-wire bridge mounted at two intermediate levels. These measurements allow us to state that stratification with a linear dependence of density on height was established in the chamber with an accuracy of 10%. The optical means of investigation of the motion of thermics include laser sheet 7 (a beam of an LG-106M laser, which was fan-shaped by a convex cylindrical mirror 8) and cine camera 9.

5. A sequence of pictures in Fig. 2 shows the motion of a single thermic in a stratified medium. The laws of its motion at the initial section practically coincide with those for thermics moving in a homogeneous medium [13]: a toroidal vortex ring floating up with a roughly constant velocity is formed from a hemispherical thermic. During thermic ascent, the radius of the vortex ring R increases in accordance with the formula [6]

$$\alpha = \frac{dR}{dz} = \frac{4\xi V g}{\Gamma^2(2\ln(8R/r) - 1)}, \tag{6}$$

where r is the small radius of the torus and Γ is the circulation of the vortex ring. The difference from the case

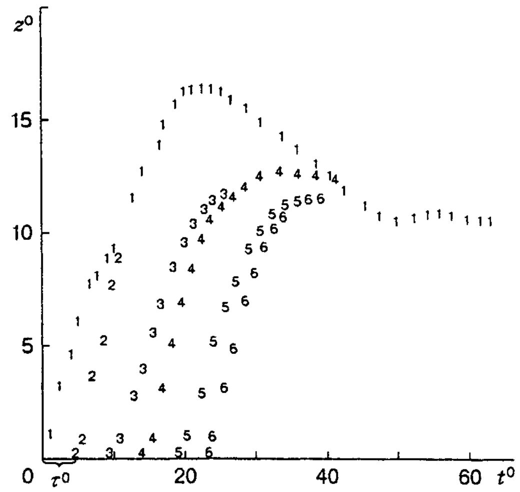

Fig. 5

of a homogeneous atmosphere is observed when the thermic approaches the line of zero buoyancy. Beginning from the level $z^0 = 6$, the parameter $\alpha = dR/dz$ (the ratio of the rate of increasing of the vortex-ring radius to the velocity of ascent of the thermic) decreases. The dependence of the parameter α on the height of thermic ascent z^0 is plotted in Fig. 3. At the level of zero buoyancy, ξ changes its sign and α becomes negative in accordance with formula (6). The ascent of the vortex ring slows down (the motion graphs are given in Fig. 4). Ascending to the height $z^0 = 1.3H_0^0$, the thermic stops; then it descends, exciting a strong internal wave, and gradually disintegrates after several decaying oscillations at the Brunt–Väisälä frequency. For example, for a thermic whose motion is plotted by the solid curve in Fig. 4 (the circles indicate the motion of the upper point of the thermic and the triangles show the motion of the lower point), the level of zero buoyancy was located at the height $H_0^0 = 16$ ($\xi = 0.5$). The dimensionless period of oscillations $T_A^0 = 2\pi\sqrt{H_0^0} = 25$ calculated from formulas (2) and (3) corresponds to the experimentally measured time between the two maxima of height in the graph of motion of this thermic.

In some cases, when the thermic passed the line of zero buoyancy, the toroidal core of the thermic split into two parts ascending with different velocities.

The motion of periodically arising thermics was studied using a rotating platform 2 (see Fig. 1) with hemispherical soap bubbles filled by a nitrogen-helium medium of given density and prepared for starting. The velocity of platform rotation was varied by changing the voltage supplied to the engine. The motion of multiple thermics initiated at one point of space with equal time intervals is plotted in Fig. 5 in dimensionless coordinates (3) (the numbers in the figure refer to the numbers of the thermics).

We note the main features of their motion. In this case, the level of zero buoyancy was located at the height $H^0 \sim 10$. Six thermics were initiated with the time interval $\tau^0 = 4$. Up to the height $z^0 = 5$, the first and second thermics rise independent of each other; then the second thermic moving in the atmosphere excited by the first thermic catches up with the latter at the height $z^0 \sim 12$ and is pulled into it — the classical *muddle of vortices* is observed [14, p. 305]; after that both thermics disintegrate, descend to the height of zero buoyancy, and distort the axial symmetry of motion. Entering the disturbed field, the motion of the third thermic slows down, not reaching the height of zero buoyancy, and the third thermic is also destroyed at the height $z^0 \sim 12$ interacting with the fourth thermic. The same occurs for the subsequent thermics. As a result, at $t^0 \sim 40$–50 all the thermics merge and form an expanding shapeless cloud at a height of approximately 70% of the height of ascent of a single thermic in the same stratified atmosphere, i.e., approximately at the level of zero buoyancy.

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